

Transport properties of anisotropically expanding quark-gluon plasma within a quasi-particle model

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The bulk and shear viscosities (η and ζ) have been studied for quark-gluon-plasma produced in relativistic heavy ion collisions within semi-classical transport theory in a recently proposed quasi-particle model of (2+1)-flavor lattice QCD equation of state. These transport parameters have been found to be highly sensitive to the interactions present in hot QCD. Contributions to the transport coefficients from both the gluonic sector and the matter sector have been investigated. The matter sector is found to be significantly dominating over the gluonic sector, in the case of both η and ζ . The temperature dependences of the quantities, ζ/S , and ζ/η indicate a sharply rising trend for the ζ closer to QCD transition temperature. Both η and ζ are shown to be equally significant for the temperatures that are accessible in the relativistic heavy ion collision experiments, and hence play crucial role while investigating the properties of the quark-gluon plasma.

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I. INTRODUCTION

The study of transport coefficients for hot QCD matter is an area of intense research since the discovery of a fluid like picture of quark-gluon-plasma (QGP) in the relativistic heavy ion collider (RHIC) at BNL [1]. The discovery of the QGP is attributed to the fact that at extreme energy-density and temperature, the ordinary nuclear matter goes through a transition to the QGP phase as predicted by the finite temperature Quantum-Chromodynamics (QCD) (this transition is shown to be a crossover [2] at the vanishing baryon density).

To describe a fluid, shear and bulk viscosities η and ζ respectively) are very important physical quantities that characterize dissipative processes in its hydrodynamic evolution. The former describes the entropy production due to the transformation of the shape of hydrodynamic system at a constant volume, and the latter describes the entropy production at the constant rate of change of the volume of the system (hot fireball at the RHIC). Moreover, η/S , and ζ/S serve as the inputs while studying the hydrodynamic evolution of the fluid [3, 4]. One can also couple hydrodynamics with the Boltzmann descriptions at the later stages after the collisions of heavy ions at the RHIC, by maintaining the continuity of the entire stress energy tensor and currents. That could be translated in terms of the viscous modifications to the thermal distributions functions of particles. This leads to a smooth transition from the hydrodynamic regime where the mean free paths are short to a region where hydrodynamics is inapplicable and Boltzmann treatments seems

to be justified [5]. Therefore, this sets a way to study the impact of transport coefficients of the QGP in various processes at the RHIC, and the ongoing heavy ion experiments at Large Hadron Collider (LHC), CERN (*e.g.* dilepton production, quarkonia physics *etc.*). Regarding viscous corrections to dilepton production rate at the RHIC, we refer the reader to [6]. The determination of η and ζ , has to be done separately from a microscopic theory, either from a transport equation [7] with an appropriate Force, Collision and Source terms or equivalently from the field theoretic approach using Green-Kubo formulas [8] (long wavelength behavior of the correlations among various components of the stress-energy tensor).

The QGP is strongly interacting at the RHIC [1], as inferred from the flow measurements, and strong jet quenching observed there. This observation is found to be consistent with the lattice simulations of the hot QCD equation of state(EOS) [9, 10], which predict a strongly interacting behavior even at temperatures which are a few T_c (the QCD transition temperature). The flow measurements suggest, a very tiny value for the ratio of η to the entropy density, S (η/S) for the QGP, and the near perfect fluid picture [11–14] (except near the QCD transition temperature where ζ/S is equally significant η/S [15–18]).

Preliminary studies at the LHC [19–21] reconfirm above mentioned observations regarding the QGP. In heavy-ion collisions at the LHC, in addition to the elliptic flow obtained at the RHIC, there are other interesting flow patterns, *viz.*, the dipolar, and the triangular flow, which are sensitive to the initial collision geometry [22]. There have been recent interesting studies to understand them at LHC [19, 23]. A more precise measurements of various flows and jet quenching at LHC is

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awaited. On the other hand, ζ has achieved considerable attention in the context of the QGP after the interesting reports on its rising value close to the QCD transition temperature [15]. Subsequently, the possible impact of the large bulk viscosity of the QGP at the RHIC have been studied by several authors; Song and Heinz [24] have studied the interplay of shear and bulk viscosities in the context of collective flow in heavy ion collisions. Their study revealed that one can not simply ignore the bulk viscosity while modeling the QGP. In this context, there are other interesting studies in the literature [25–31]. The role of bulk viscosity in freeze out phenomenon has been offered in [21, 32]. Effects of bulk viscosity in hadronic phase, and in the hadron emission has been studied in [33]. Interestingly, in the recent investigations, these transport coefficients are found to be very sensitive to the interactions [13, 14], and nature of the phase transition in QCD [34]. Another crucial aspect of ζ is its influence on the domain of applicability of hydrodynamics at the RHIC, *viz.* the phenomenon cavitation. This phenomenon has been addressed in detail in the context of diverging value of ζ near the QCD transition temperature in [35, 36]. Thus, the determination of η and ζ for the QGP has multi-facet dimensions, and impact on the variety of physical phenomena at the RHIC and the LHC.

The determination of η and ζ has been performed adopting the view-point based on the inference drawn from the experimental results, and the lattice QCD (the best known non-perturbative technique to address the QGP). Lattice QCD has indeed been very successful to study the QGP thermodynamics. However, the computation of transport coefficients in lattice QCD is a very non-trivial exercise, due to several uncertainties and inadequacy in their determination. Despite that there are a few first results computed from lattice QCD for bulk and shear viscosities [37–40] which have observed the small value of η/S , and large ζ/S at the RHIC. A very recent interesting analysis [41] suggests that it is possible to compare the direct lattice results with the experiments at the RHIC. From such a comparison the QCD transition temperature came out to be around 175MeV . More refined lattice studies on η and ζ are awaited in the near future.

The work presented in this paper is an attempt to achieve, (i) temperature dependence of η and ζ (The gluonic as well as the matter sector contributions to these transport parameters have been obtained by combining a transport equation with a recently proposed quasi-particle model [42–44] of $(2+1)$ -flavor lattice QCD EOS. Noteworthy point is that the matter sector has largely been ignored in the literature in this context), (ii) to understand the small η/S , and large ζ/S for the QGP for the temperatures closer to T_c . More precisely, inputs has been taken from the computations of η and ζ in quasi-particle models [13, 14, 18, 45, 46], and combine the understanding with a transport theory determination of them in the presence of Chromo-Weibel instabil-

ities [12, 47, 48]. The present work is the extension of our recent work on η [13, 14], and ζ [18] for the gluonic sector, to the full QCD.

The paper is organized as follows. In Sec. II, we present the formalism to compute the η and ζ . The quasi-particle model and transport equation have also been discussed in brief in the same section. In Sec. III, we have presented the results on the temperature dependence of η and ζ in $(2+1)$ -flavor lattice QCD, and relevant physics. In Sec. IV, we have presented conclusions and future prospects of the work.

II. DETERMINATION OF TRANSPORT COEFFICIENTS

There may be a variety of physical phenomena which can lead to viscous effects in the QGP (or in general any interacting system) [5]. Among them, particular focus in on the viscous effects which get contributions from the the classical chromo-fields.

The idea adopted here is based on the mechanism earlier proposed [12, 48] to explain the small viscosity of a weakly coupled, but expanding QGP. The mechanism in the context of the QGP is solely based on the particle transport processes in the turbulent plasmas [49] that are characterized by strongly excited random field modes in the certain regimes of instability. They coherently scatter the charged particles and thus reduce the rate of momentum transport. This eventually lead to the suppression of the transport coefficients in plasmas. This phenomenon has been studied both in electro-magnetic (EM) plasmas [50], and in non-abelian plasmas (QCD plasma) by Asakawa, Bass and Müller [12, 48], and further employed for the realistic QGP EOS in [13, 14].

The condition for the spontaneous formation of turbulent fields can be achieved in EM plasmas with an anisotropic momentum distribution [51] of charged particles, and in the QGP with anisotropic distribution of thermal partons [52]. In the context of pure $SU(3)$ gauge theory, this mechanism turn out to be successful to explain small shear viscosity of the QGP and larger bulk viscosity for the temperatures accessible at the RHIC and the LHC [14, 18]. Here, extension has been desired to the case of realistic EOS for the QGP by incorporating the effects from the matter sector (quark-antiquarks).

It will be seen later that the analysis leads to an interesting observation regarding the relative contribution of the gluonic and matter sector to the transport parameters. Before, we give a brief description of the quasi-particle understanding of $(2+1)$ -lattice QCD that furnishes an appropriate modeling of equilibrium state.

A. The quasi-particle description of hot QCD

Quasi-particle description of hot QCD medium effects, is not a new concept. There has been several attempts

so far to understand hot QCD medium effects in terms of non-interacting/weakly interacting quasi-partons, *viz.*, the effective thermal mass models[53, 54], the effective mass model with temperature dependent bag parameter to cure the problem of thermodynamic inconsistency[54], the effective quasi-particles with gluon condensate[55], the Polyakov loop models[56] (Polyakov loop acts as effective fugacity), and the quasi-partons with effective fugacities [42–44]. The last one, that will be employed here, shown to be fundamentally distinct from all other mentioned models, and in the spirit of Landau’s theory of Fermi liquids. Moreover, the model has been highly successful, in interpreting the lattice QCD thermodynamics, and bulk and transport properties of hot QCD matter and the QGP in relativistic heavy ion collisions.

In our quasi-particle description for (2+1)-flavor lattice QCD[44], we start with the ansatz that the Lattice QCD EOS can be interpreted in terms of non-interacting quasi-partons having effective fugacities, z_g, z_q which encodes all the interaction effects, where z_g denotes the effective gluon fugacity, and z_q denotes the effective quark-fugacity respectively. In this approach the hot QCD medium is divided in to two sectors, *viz.*, the effective gluonic sector, and the matter sector (light quark sector, and strange quark sector). The former refers to the contribution of gluonic action to the pressure which also involves contributions from internal fermion lines. On the other hand, latter involve interactions among quark, anti-quarks, as well as their interactions with gluons. The ansatz can be translated to the form of the equilibrium distribution functions, f_{eq}^g , f_{eq}^q , and f_{eq}^s as follows,

$$\begin{aligned} f_{eq}^g &= \frac{z_g \exp(-\beta p)}{\left(1 - z_g \exp(-\beta p)\right)}, \\ f_{eq}^q &= \frac{z_q \exp(-\beta p)}{\left(1 + z_q \exp(-\beta p)\right)}, \\ f_{eq}^s &= \frac{z_q \exp(-\beta \sqrt{p^2 + m^2})}{\left(1 + z_q \exp(-\beta \sqrt{p^2 + m^2})\right)}, \end{aligned} \quad (1)$$

where m denotes the mass of the strange quark, which we choose to be $0.1 GeV$. $\beta = T^{-1}$ denotes inverse of the temperature. Here, we are working in the units where Boltzmann constant, $K_B = 1$, $c = 1$, and $\hbar/2\pi = 1$. We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 2$ for light quarks, $\nu_s = 2 \times 2 \times N_c \times 1$ for the strange quark for $SU(N_c)$. Here, we are dealing with $SU(3)$, so $N_c = 3$. Since the model is valid in the deconfined phase of QCD (beyond T_c), therefore, the mass contributions of the light quarks can be neglected as compared to the temperature. Therefore, in our model, we only consider the mass for the strange quarks.

The effective fugacity is not merely a temperature dependent parameter which encodes the hot QCD medium

effects. It is very interesting and physically significant. The physical significance reflects in the modified dispersion relation both in the gluonic and matter sector. In this description, the effective fugacities modify the single quasi-parton energy as follows,

$$\begin{aligned} \omega_p^g &= p + T^2 \partial_T \ln(z_g) \\ \omega_p^q &= p + T^2 \partial_T \ln(z_q) \\ \omega_p^s &= \sqrt{p^2 + m^2} + T^2 \partial_T \ln(z_q). \end{aligned} \quad (2)$$

These dispersion relations can be explicated as follows. The single quasi-parton energy not only depends upon its momentum but also gets contribution from the collective excitations of the quasi-partons. The second term is like the gap in the energy-spectrum due to the presence of quasi-particle excitations. This makes the model more in the spirit of the Landau’s theory of Fermi -liquids. A detailed discussions of the interpretation and physical significance of z_g , and z_q , we refer the reader to our recent work [44]. Henceforth, we shall use gluon sector in the place of effective gluonic sector for the sake of ease. We shall now proceed to the determination η and ζ in presence of Chromo-Weibel instabilities.

B. Chromo-Weibel instability and the anomalous transport

The determination of η and ζ have been done in a multi-fold way. Firstly, we need an appropriate modeling of distribution function for the equilibrium state. Secondly, we need to set up an appropriate transport equation to determine the form of the perturbation to the distribution function. These two steps eventually determine these transport coefficients. For the former step, we employ the quasi-particle model for the (2+1)-flavor lattice QCD EOS discussed earlier.

The η and ζ has two contributions same as the shear viscosity in [12], (i) from the Vlasov term which captures the long range component of the interactions, and (ii) the collision term, which models the short range component of the interaction. Here, we shall only concentrate on the former case. The determination of shear and bulk viscosities from an appropriate collision term will be a matter of future investigations. Importantly, the analysis adopted here is based on weak coupling limit in QCD, therefore, the results are shown beyond $1.2T_c$ assuming the validity of weak coupling results for the QGP there. Note that the interplay for anomalous and collisional components of η has been discussed in [12–14], and in the case of ζ for the pure gauge theory, a discussion has been presented regarding the interplay of the collisional [57–59], and anomalous components in [18]. It seems that at the conceptual level all the observation in [18] regarding the interplay will remain valid here. Since, we do not have results for the matter sector therefore, we shall not offer a quantitative discussions on such an inter-play here. There have been computations of transport parameters in

the case of pure gauge theory based on the effective mass models within relaxation time approximation [60]. The approach adopted, and the physics set-up is entirely distinct in the present case. It is to be noted that the gluonic component in all the quantities is denoted by subscript g , light-quark components are denoted by subscript q , and strange quark component is denoted by subscript s .

C. Determination of ζ and η

Let us first briefly outline the standard procedure of determining transport coefficients in transport theory [7, 12]. The bulk and shear viscosities, ζ and η of the QGP in terms of equilibrium parton distribution functions are obtained by comparing the microscopic definition of the stress tensor with the macroscopic definition of the viscous stress tensor. The microscopic definition of the stress tensor is as,

$$T_{ik} = \int \frac{d^3p}{(2\pi)^3 E_p} p_i p_k f(\vec{p}, \vec{r}). \quad (3)$$

On the other hand, macroscopic expression for the viscous stress tensor is given by,

$$T_{ik} = P\delta_{ik} + \epsilon u_i u_k - 2\eta(\nabla u)_{ik} - \zeta\delta_{ik}\nabla \cdot \vec{u}, \quad (4)$$

where $(\nabla u)_{ik}$ is the traceless, symmetrized velocity gradient, and $\nabla \cdot \vec{u}$ is the divergence of the fluid velocity field. E_p accounts for the dispersion relation. To determine ζ and η , one writes the parton distribution functions as

$$f(\vec{p}, \vec{r}) = \frac{1}{z_{g/q} \exp(-\beta u \cdot p + f_1(\vec{p}, \vec{r})) \mp 1}. \quad (5)$$

Assuming that $f_1(\vec{p}, \vec{r})$ is a small perturbation to the equilibrium distribution, we expand $f(\vec{p}, \vec{r})$ and keep the linear order term in f_1 ; this leads to,

$$\begin{aligned} f(\vec{p}, \vec{r}) &= f_0(\mathbf{p}) + \delta f(\vec{p}, \vec{r}) \\ &= f_0(\mathbf{p}) \left(1 + f_1(\vec{p}, \vec{r})(1 \pm f_0(\mathbf{p})) \right), \end{aligned} \quad (6)$$

where $f_0 \equiv \{f_g, f_q, f_s\}$, and similarly $f_1 \equiv \{f_1^g, f_1^q, f_1^s\}$. The *plus* sign in the bracket is for gluons, and *minus* sign is for fermions (q and s). Next, we shall consider these quantities explicitly in the gluonic and the matter sectors. As discussed in [12, 14], ζ and η are determined by taking the following form of the perturbation f_1 ,

$$f_1^g(\vec{p}, \vec{r}) = -\frac{1}{\omega_g T^2} p_i p_j \left(\Delta_{1g}(p) \nabla u_{ij} + \Delta_{2g}(\vec{p}) \delta_{ij} \right)$$

$$\begin{aligned} f_1^q(\vec{p}, \vec{r}) &= -\frac{1}{\omega_q T^2} p_i p_j \left(\Delta_{1q}(p) \nabla u_{ij} + \Delta_{2q}(\vec{p}) \delta_{ij} \right), \\ f_1^s(\vec{p}, \vec{r}) &= -\frac{1}{\omega_s T^2} p_i p_j \left(\Delta_{1s}(p) \nabla u_{ij} + \Delta_{2s}(\vec{p}) \delta_{ij} \right). \end{aligned} \quad (7)$$

Here the dimensionless functions $\Delta_{1g,1q,1s}(p), \Delta_{2g,2q,2s}(\vec{p})$ measure the deviation from the equilibrium configuration. $\Delta_1(p), \Delta_2(\vec{p})$, lead to η and ζ respectively. Note that $\Delta_{1g,1q,1s}(p)$ is a isotropic function of the momentum in contrast to $\Delta_{2g,2q,2s}(\vec{p})$, which is anisotropic in momentum \vec{p} .

Since ζ and η are Lorentz scalars; they may be evaluated conveniently in the local rest frame of the fluid (in the local rest frame $f_0 \equiv f_{eq}$). Considering the a boost invariant longitudinal flow, $\nabla \cdot u = \frac{1}{\tau}$ and, $(\nabla u)_{ij} = \frac{1}{3\tau} \text{diag}(-1, -1, 2)$ in the local rest frame. In this case, the perturbations, $f_1(p)$ take the form,

$$\begin{aligned} f_1^g(\vec{p}) &= -\frac{\Delta_{1g}(p)}{\omega_g T^2 \tau} \left(p_z^2 - \frac{p^2}{3} \right) - \frac{\Delta_{2g}(\vec{p})}{\omega_g T^2 \tau} p^2 \\ f_1^q(\vec{p}) &= -\frac{\Delta_{1q}(p)}{\omega_q T^2 \tau} \left(p_z^2 - \frac{p^2}{3} \right) - \frac{\Delta_{2q}(\vec{p})}{\omega_q T^2 \tau} p^2, \\ f_1^s(\vec{p}) &= -\frac{\Delta_{1s}(p)}{\omega_s T^2 \tau} \left(p_z^2 - \frac{p^2}{3} \right) - \frac{\Delta_{2s}(\vec{p})}{\omega_s T^2 \tau} p^2. \end{aligned} \quad (8)$$

where τ is the proper time ($\tau = \sqrt{t^2 - z^2}$). The shear viscosities are obtained in terms of entirely unknown functions, $\Delta_{1g,1q,1s}(p)$ as,

$$\begin{aligned} \eta_g &= \frac{\nu_g}{15T^2} \int \frac{d^3p}{8\pi^3} \frac{p^4}{\omega_g^2} \Delta_{1g}(p) f_g(1 + f_g) \\ \eta_q &= \frac{\nu_q}{15T^2} \int \frac{d^3p}{8\pi^3} \frac{p^4}{\omega_q^2} \Delta_{1q}(p) f_q(1 - f_q) \\ \eta_s &= \frac{\nu_s}{15T^2} \int \frac{d^3p}{8\pi^3} \frac{p^4}{\omega_s^2} \Delta_{1s}(p) f_s(1 - f_s) \end{aligned} \quad (9)$$

The bulk viscosities are obtained in terms of the unknown functions, $\Delta_{2g,2q,2s}(\vec{p})$,

$$\begin{aligned} \zeta_g &= \frac{\nu_g}{3T^2} \int \frac{d^3p}{8\pi^3} \frac{p^2}{\omega_g^2} (p^2 - 3c_s^2 E_p^2) \Delta_{2g}(\vec{p}) f_g(1 + f_g) \\ \zeta_q &= \frac{\nu_q}{3T^2} \int \frac{d^3p}{8\pi^3} \frac{p^2}{\omega_q^2} (p^2 - 3c_s^2 \omega_q^2) \Delta_{2q}(\vec{p}) f_q(1 - f_q) \\ \zeta_s &= \frac{\nu_s}{3T^2} \int \frac{d^3p}{8\pi^3} \frac{p^2}{\omega_s^2} (p^2 - 3c_s^2 \omega_s^2) \Delta_{2s}(\vec{p}) f_s(1 - f_s) \end{aligned} \quad (10)$$

Notice that while obtaining the expression for the bulk viscosity, we have exploited the Landau-Lifshitz Condition for the stress energy tensor. The factor $(-3c_s^2 E_p^2)$ in the right-hand side of Eq.(10) is coming only because of that. For details, we refer the reader to [45]. Here, c_s^2 is the speed of sound square extracted from the lattice data on $(2+1)$ -flavor lattice QCD. The determination of $\Delta_{1g,1q,1s}(p)$, and $\eta_{g,q,s}$ can easily be done following [13, 14], and $\Delta_{2g,2q,2s}(\vec{p})$ and $\zeta_{g,q,s}$ following [18].

D. Determination of the perturbative, Δ_1 and Δ_2

To obtain a analytic expression for the perturbations, $\Delta_{1,2}$, in our analysis, one need to first set up the transport equation in the presence of turbulent color fields. This has been done in [12–14] in the recent past. Here, we only quote the linearized transport equation, with Vlasov-Dupree diffusive term, which arise after considering the ensemble average over turbulent color fields, in the light-cone frame. The transport equation thus obtained reads,

$$v^\mu \frac{\partial}{\partial x^\mu} f_{eq}(p) + \mathbf{V}_A f_1 f_{eq}(p)(1 \pm f_{eq}(p)) = 0, \quad (11)$$

where $f_{eq} \equiv \{f_g, f_q, f_s\}$, and $v^\mu = \hat{p}$ (the quasi-particle model does not change the group velocity of the quasi-partons). Note that Eq. (11) is written in the absence of collision term, and assuming the weak coupling approximation.

The mathematical structure of the Vlasov-Dupree operator is as follows,

$$\mathbf{V}_A = \frac{g^2 C_2}{2(N_c^2 - 1)\omega^2} < E^2 + B^2 > \tau_m \mathbf{L}^2, \quad (12)$$

where C_2 is the quadratic Casimir invariant for partons. For gluons, $C_2 = N_c$, and for quarks $C_2 = (N_c^2 - 1)/2N_c$. Here, $\omega \equiv \{\omega_g, \omega_q, \omega_s\}$, denotes the quasi-partons dispersions. The parameter, τ_m is the time scale associated with instability in the field, and the operator \mathbf{L}^2 is,

$$\begin{aligned} \mathbf{L}^2 &= -(\vec{p} \times \partial_{\vec{p}})^2 + (\vec{p} \times \partial_{\vec{p}})|_z^2 \\ &\equiv -(L^p)^2 + (L^p_z)^2. \end{aligned} \quad (13)$$

Since \mathbf{L}^2 contains angular momentum operator L^p , therefore it gives non-vanishing contribution while operating on a anisotropic function of \vec{p} . It will always lead to the vanishing contribution while operating on a isotropic function of \vec{p} . Following [14], the expression for the $\Delta_{1g}(p)$ is obtained as,

$$\Delta_{1g}(p) = \frac{2(N_c^2 - 1)\omega_g^2 T}{3C_g g^2 < E^2 + B^2 > \tau_m}. \quad (14)$$

On the other hand, expressions for $\Delta_{1q,1s}$ are obtained as,

$$\begin{aligned} \Delta_{1q}(p) &= \frac{2(N_c^2 - 1)\omega_q^2 T}{3C_f g^2 < E^2 + B^2 > \tau_m} \\ \Delta_{1s}(p) &= \frac{2(N_c^2 - 1)\omega_s^2 T}{3C_f g^2 < E^2 + B^2 > \tau_m}. \end{aligned} \quad (15)$$

Now, we write the transport equation containing only those terms which contribute to bulk viscosity ζ as,

$$\begin{aligned} &(\frac{p^2}{3\omega^2} - c_s^2) \frac{\omega}{T} (\nabla \cdot \vec{u}) f_{eq}(1 \pm f_{eq}) \\ &= \frac{g^2 C_2}{3(N_c^2 - 1)\omega^2} < E^2 + B^2 > \tau_m \mathbf{L}^2 f_1(\vec{p}, \vec{r}) f_{eq}(1 \pm f_{eq}) \end{aligned}$$

Following, [18], we can obtain the mathematical forms of the corresponding perturbations, Δ_2 . We shall write down the expressions in the gluonic sector, and matter sector separately to avoid the confusion. The expression for $\Delta_{2g}(p)$ is obtained as,

$$\Delta_{2g}(\vec{p}) = \frac{4(N_c^2 - 1)T\omega_g^2}{N_c g^2 < E^2 + B^2 > \tau_m p^2} (\frac{p^2}{3} - c_s^2 \omega_g^2) \ln(\frac{p_T}{\sqrt{6}T}) \quad (17)$$

On the other hand, the expressions for $\Delta_{2q,2s}$ are obtained as,

$$\begin{aligned} \Delta_{2q}(\vec{p}) &= \frac{4(N_c^2 - 1)T\omega_q^2}{C_f g^2 < E^2 + B^2 > \tau_m p^2} (\frac{p^2}{3} - c_s^2 \omega_q^2) \ln(\frac{p_T}{\sqrt{6}T}) \\ \Delta_{2s}(\vec{p}) &= \frac{4(N_c^2 - 1)T\omega_s^2}{C_f g^2 < E^2 + B^2 > \tau_m p^2} (\frac{p^2}{3} - c_s^2 \omega_s^2) \ln(\frac{p_T}{\sqrt{6}T}). \end{aligned} \quad (18)$$

Here, $C_f \equiv \frac{N_c^2 - 1}{2N_c}$.

Next, we relate the denominator of Eqs.(15), (17), and (18) to the parton energy loss parameter $\hat{q} \equiv \hat{q}_g, \hat{q}_q$, via the relation,

$$\hat{q} = \frac{2g^2 N_c}{3(N_c^2 - 1)} < E^2 + B^2 > \tau_m. \quad (19)$$

Now the gluonic, contributions to η , and ζ in terms of \hat{q} can be rewritten as follows,

$$\begin{aligned} \eta_g &= \frac{T^6}{\hat{q}} \frac{64(N_c^2 - 1)}{3\pi^2} PolyLog[6, z_g], \\ \zeta_g &= \frac{4(N_c^2 - 1)}{3T\pi^2 \hat{q}} \int \int p_T dp_T dp_z (\frac{p^2}{3} - c_s^2 \omega_g^2)^2 \times \\ &\quad \ln(\frac{p_T}{p_0}) \times f_g(1 + f_g). \end{aligned} \quad (20)$$

On the other hand, quark-antiquark viscosities in the matter sector are obtained as,

$$\begin{aligned} \eta_q &= \frac{64N_c^2 \nu_q}{3\pi^2 \hat{q}(N_c^2 - 1)} \{-PolyLog[6, -z_q]\} \\ \eta_s &= \frac{64N_c^2 \nu_s}{3\pi^2 \hat{q}(N_c^2 - 1)} \left\{ -PolyLog[6, -z_q] \right. \\ &\quad \left. + \frac{\tilde{m}^2}{2} PolyLog[5, -z_q] \right\} \\ \zeta_{q,s} &= \frac{N_c \nu_{q,s}}{3C_f T \pi^2 \hat{q}} \int \int p_T dp_T dp_z (\frac{p^2}{3} - c_s^2 \omega_{q,s}^2)^2 \times \\ &\quad \ln(\frac{p_T}{p_0}) \times f_{q,s}(1 - f_{q,s}). \end{aligned} \quad (21)$$

The $PolyLog$ functions that appear in the expressions for $\eta_{g,q,s}$ are defined in terms of the series representation as,

$$Polylog[n, x] = \sum_{k=1}^{\infty} \frac{x^k}{k^n}, \quad (22)$$

where n is a positive integer, and the convergence of the series is ensured by the fact that $x \leq 1$. Moreover, $(16) PolyLog[n, 1] \equiv \zeta(n)$, and also $PolyLog[n, -1] \sim -\zeta(n)$.

III. TEMPERATURE DEPENDENCE OF η AND ζ

The determination of η , and ζ in the gluonic and matter sector, is incomplete unless to fix the temperature dependence of \hat{q} in both the sectors. The determination of \hat{q} has been presented in the various phenomenological studies [61], either based on the eikonal approximation, or the higher twist approximation, at a particular value of the temperature. Here, we choose the \hat{q} for gluons as $4.5 \text{ GeV}^2/\text{fm}$, and $2.0 \text{ GeV}^2/\text{fm}$ for quarks, at $T = 0.4 \text{ GeV}$ [62] (this temperature, we denote as T_0). Since, \hat{q} appears in the denominator in the expressions for η and ζ . Therefore, any set of values higher than those mentioned above will further decrease the values of η and ζ . At $T = T_0$, we can see that $\hat{q}_g = 2.25\hat{q}_q$. At this juncture, we do not know these parameters at all temperatures, so we assume this relation holds for all temperatures. This assumption is based on definition of \hat{q} in the leading order in hot QCD [63], where it is same for both gluons and quarks except that of the quadratic Casimir factor. We shall utilize the relation $\hat{q}_g = 2.25\hat{q}_q$, while studying the temperature dependence of various quantities in the next subsections. The exact temperature dependence of \hat{q} , employing the quasi-particle description of hot QCD is not known to us at the moment. This will be a matter of future investigations.

A. Relative contributions

In the section, discussions are mainly on, (i) relative contributions of various components of η with their ideal counterparts, (ii) gluonic versus matter sector for η , and ζ .

Note that the shear and bulk viscosities in the full QCD can be obtained by summing of all the individual contributions of the quasi-partons as,

$$\begin{aligned}\eta &= \eta_g + \eta_q + \eta_s \\ \zeta &= \zeta_g + \zeta_q + \zeta_s.\end{aligned}\quad (23)$$

The additivity of various components here is attributed to the fact that all of them belong to same process *viz.* the anomalous transport. Viscosity contributions from distinct process (e.g. anomalous and collisional) are inverse additive due to the fact that various rates [12, 18] are additive.

Let us define the relative quantities of our interest. Firstly, we shall define the ratios of various components of η to that for the ideal system of quarks and gluons (denoted as η^{Id}), which are defined as follows,

$$\begin{aligned}R_{gi} &\equiv \frac{\eta_g}{\eta_g^{Id}} \\ R_{qi,si} &\equiv \frac{\eta_{q,s}}{\eta_{q,s}^{Id}} \\ R_i &\equiv \frac{(\eta_g + \eta_q + \eta_s)}{(\eta_g^{Id} + \eta_q^{Id} + \eta_s^{Id})}.\end{aligned}\quad (24)$$

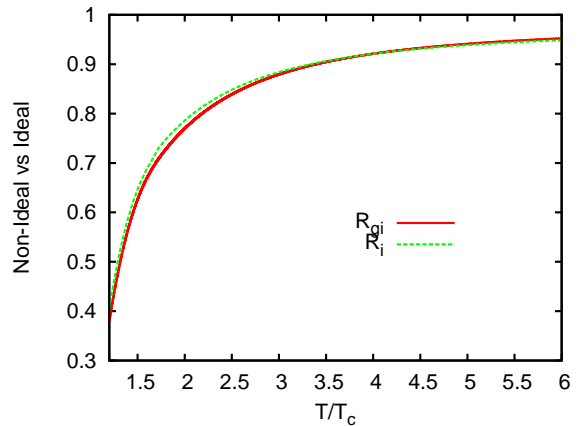


FIG. 1. (Color online) η relative to the that obtained using the ideal EOS for QGP, in the gluonic sector, and the full QCD is plotted as a function of T/T_c . The solid curve denotes the gluonic sector and dashed line denotes the full QCD. Both R_{gi} and R_i approach to the ideal limit asymptotically.

Similarly, to compare the relative contributions among various components of η , we define the following ratios,

$$\begin{aligned}R_{gq} &\equiv \frac{\eta_g}{\eta_q} \\ R_{gs} &\equiv \frac{\eta_g}{\eta_s} \\ R_{sq} &\equiv \frac{\eta_s}{\eta_q}.\end{aligned}\quad (25)$$

On the other hand, to compare the relative contributions among the various components of ζ , following quantities have been defined,

$$\begin{aligned}R^{gq} &\equiv \frac{\zeta_g}{\zeta_q} \\ R^{gs} &\equiv \frac{\zeta_g}{\zeta_s} \\ R^{sq} &\equiv \frac{\zeta_s}{\zeta_q}.\end{aligned}\quad (26)$$

The quantities defined in Eqs. (24-26) have been shown as a functions of T/T_c , in Figs. 1-4. The ratios R_{gi} and R_i are shown as a function of temperature in Fig. 1. The parameter \hat{q} assumed to be same in the interacting and ideal sector. We have considered temperature dependence beyond $1.2T_c$. Both R_{gi} , and R_i show that interactions significantly modify the shear viscosity in the gluonic sector and the full QCD at lower temperatures. Both of them lie within the range $\{0.40, 0.97\}$ for the temperature range, $\{T/T_c = 1.2, 6.0\}$. R_{qi} and R_{si} are shown in Fig. 2 as a function of temperature. Both of them sit at the top of each other. This is not surprising

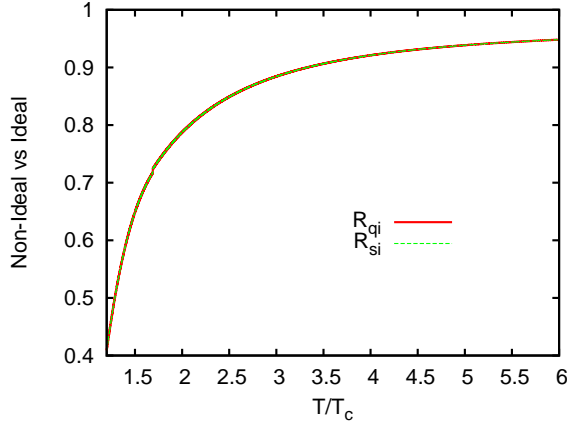


FIG. 2. (Color online) η relative to the that obtained using the ideal EOS for the QGP, in the matter sector. The R_{qi} is η relative to η^{Id} in the light-quark sector, and similarly R_{si} is for the strange quark sector. Both the curves sits on the top of each other since the mass effects from the strange-quark sector do not play significant role here.

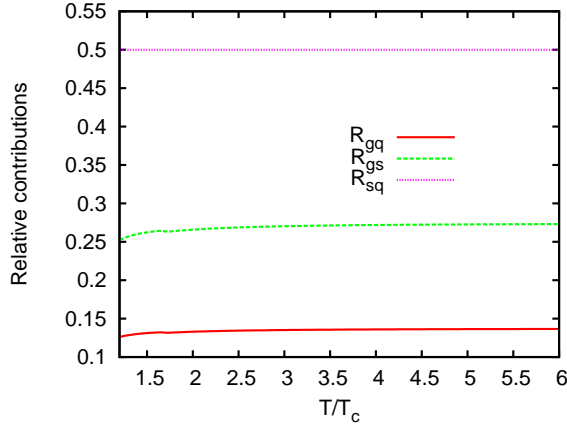


FIG. 3. (Color online) Shear viscosity in effective gluonic sector relative to matter sector. The solid lines denotes η_g relative to η_q , thin dashed lines (middle) represents η_g relative to η_s , and upper thick dashed line represents η_s relative to η_q , as a function of T/T_c .

since the mass effects coming from the strange quark sector contributes negligibly in the temperature range considered here. The light quark sector and strange quarks differ with each other by a factor of two coming from the degrees of freedom. While considering the ratio R_{si} it cancels from the numerator and denominator. From Fig.2, it evident that the hot QCD interactions significantly modify the shear viscosity in the matter sector same as in the gluonic sector as compared to the ideal counter part. All of them approaches asymptotically to the ideal limit which is nothing but 'unity'. These ob-

servations suggest that η could be thought of as a good diagnostic tool to distinguish various equations of state at RHIC and LHC.

Next, we investigate the gluonic shear and bulk viscosities relative to that of the matter sector. The relevant quantities in this context of η are R_{gq} , R_{gs} , and R_{sq} given in Eq.(25). These are shown as a function of temperature in Fig. 3. On the other hand for ζ , R^{gq} , R^{qs} , and R^{sq} are shown as a function of temperature in Fig. 4. It can be observed from Fig. 3, and Fig. 4 that the matter sector contributions significantly dominates over the gluonic contributions as far as the η and ζ are concerned. This could perhaps be understood by the following facts, *viz.*, the higher transport rates in the gluonic sector as compared to quark sector as encoded in \hat{q} , and the interactions entering through the effective fugacities z_g and z_q . Quantitatively, η_g is $\sim 0.125\eta_q$, and $0.250\eta_s$ at $T = 1.20T_c$, and increase quite slowly as a function of T/T_c reaching around $0.135\eta_q$ around $6T_c$ (see Fig. 3). The η_s almost stays $0.5\eta_q$ for the considered range of temperature (contribution from the strange quark mass is almost negligible). From Fig. 4, it can be observed that R^{gq} and R^{gs} have same qualitative behavior as a function of temperature. The quantitative difference is because of a factor ~ 2 , since $\zeta_s \sim 0.5\zeta_q$. Again the mass effects in the strange quark-sector play almost negligible role. The ratio R^{gq} initially increases and attains a peak around $T/T_c \sim 1.37$ and then decreases sharply until $T/T_c = 1.6$ and slightly increases beyond 1.6 and indicating towards the saturation at higher temperatures. Quantitatively, $\zeta_g \approx 0.27\zeta_q$ around $1.2T_c$, and $0.13\zeta_q$ at $3.0T_c$. These observations are very crucial in deciding the temperature dependence of η and ζ , and the ratios η/S , ζ/S and ζ/η . Most of the recent studies devoted to the η and ζ draw inferences for the QGP which are purely based on the study of pure $SU(3)$ sector of QCD only. The matter sector has largely been ignored. In the light of the above observations, it is not desirable to exclude the matter sector since the dominant contributions are from there. Finally, we can obtain the exact value of the ratio η/S , and ζ/S by employing the values of \hat{q} quoted earlier ($\hat{q} = 4.5\text{GeV}^2/fm$ for gluons and $2.0\text{GeV}^2/fm$ for quarks at $T = 400\text{MeV}$). η/S thus obtained as 0.570 and ζ/S came out to be 0.057 at $T = 400\text{MeV}$. As discussed earlier, to obtain the exact temperature dependence of η , and ζ , one requires to fix the temperature dependence of \hat{q} within the quasi-particle model employed here. This will be taken up separately in the near future. The quantity which can be determined unambiguity is the ratio ζ/η which is very crucial in deciding when the hot QCD becomes conformal. In other words, until what value of the temperature the effects coming from ζ are important while studying the QGP. We shall now proceed to discuss these issues next.

B. The ratio ζ/η

The behavior of the ratio ζ/η as a function of temperature is shown in Fig. 5, and the temperature dependence of the ratios $\frac{\eta}{S} \equiv \frac{\eta_{\text{qp}}}{T^3 S}$ and $\frac{\zeta}{S} \equiv \frac{\zeta_{\text{qp}}}{T^3 S}$, is shown in Fig. 6.

Most importantly, from Fig. 5, clear indications are observed that the ζ in the gluonic sector, and full QCD diverge as we approach closer to T_c (the results are not shown around T_c , since such a quasi-particle picture may not be valid there.). The quantity ζ/η shows sharp decrease until one reaches up to $1.4T_c$ in the gluonic sector and $1.6T_c$ in the full QCD sector. Beyond that the decrease becomes slow and the ratio slowly approaches to zero. Such a behavior of ζ/η as a function of temperature could mainly be described in the formal expressions in Eqs. (20), and (21), and decided only by the temperature dependence of c_s^2 but also by the energy-dispersion relations, $\omega_{g,q,s}$, and the temperature dependence of the effective fugacities, $z_{g,q}$. It is evident that there is no way to obtain a $(c_s^2 - \frac{1}{3})^2$ factor out from the expression while performing the integration. However, such a scaling could be realized whenever $p \ll T^2 \partial_T (\ln(z_{g,q}))$, and $\omega_{g,q,s}$ happen to be independent of z_g and z_q , and the thermal distribution of quasi-particles show near ideal behavior. It may perhaps be realized at a very high temperature which are not relevant to study the QGP in the RHIC and the LHC. Therefore, ζ/η obtained here does not follow either a quadratic scaling or a linear scaling with the conformal measure $(c_s^2 - \frac{1}{3})$. The same conclusions were obtained in the case of pure gauge theory recently [18]. Note that for the scalar field theories, $\zeta/\eta = 15(c_s^2 - \frac{1}{3})^2$ (quadratic scaling) [64], and it has been found to be true for a photon gas coupled with the matter [65]. The quadratic scaling is also valid in the case of perturbative QCD with a proportionality factor different from 15 [66]. Furthermore, in the case of near conformal theories with gravity dual, ζ/η shows linear dependence on $(c_s^2 - \frac{1}{3})$ [67].

Finally, in Fig. 6, $\frac{\eta}{S}$ and $\frac{\zeta}{S}$ are plotted as a function of T/T_c . For entropy density, we utilize the quasi-particle results which are shown to be consistent with the predictions of lattice QCD, and in all the plots c_s^2 has been obtained from the quasi-particle model employing the method quoted in [68]. Interestingly, these are of same order at $T = 1.2T_c$. Below that temperature the latter dominates over the former and vice versa for $T \geq 1.2T_c$. The former increases, in contrast to latter as a function of T/T_c . There is a sharp increase shown by the latter until one reaches $1.4T_c$, and beyond that the decrease is slower and one is quite close to the conformal limit of QCD. The important inference that could be drawn from here is that while studying the QGP one need to incorporate the effects of both shear and bulk viscosity until approximately $1.5T_c$. This confirms our viewpoint that both η and ζ have significant impact on the properties of the QGP at the RHIC and the LHC.

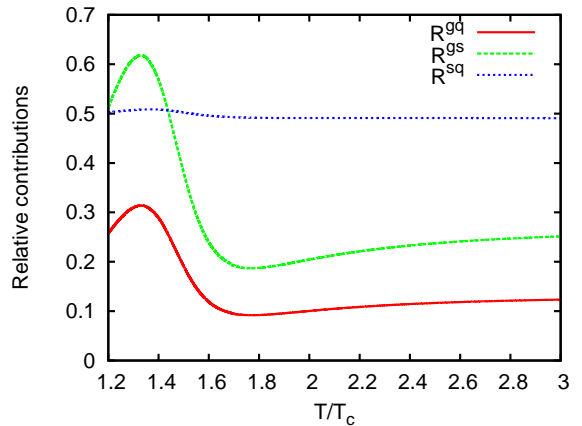


FIG. 4. (Color online) Temperature dependence of relative quasi-particle bulk viscosities. The solid line shows the behavior of ζ_g relative to ζ_q , thin dashed line shows the behavior of ζ_g relative to ζ_s , and the thick dashed line shows the behavior of ζ_s relative to ζ_q , as a function of T/T_c .

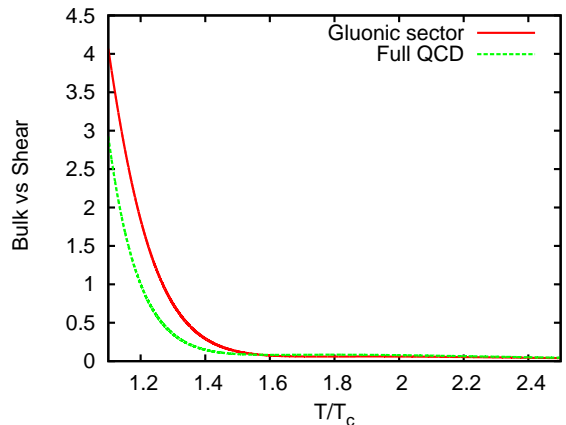


FIG. 5. (Color online) Temperature dependence of the ratio ζ/η , in the effective gluonic sector, and the full QCD. The solid line represents the gluonic sector, and the dashed line represents the full QCD.

IV. CONCLUSIONS AND FUTURE PROSPECTS

In conclusion, shear and bulk viscosities of hot QCD are estimated by combining a semi-classical transport equation with a quasi-particle realization of (2+1)-flavor lattice QCD. The effective gluonic sector contributes an order of magnitude lower as compared to the matter sector while determining the transport coefficients of hot QCD and the QGP. This could perhaps be understood in terms of transport cross-sections of gluons and quark-antiquarks. Since transport coefficients are inversely pro-

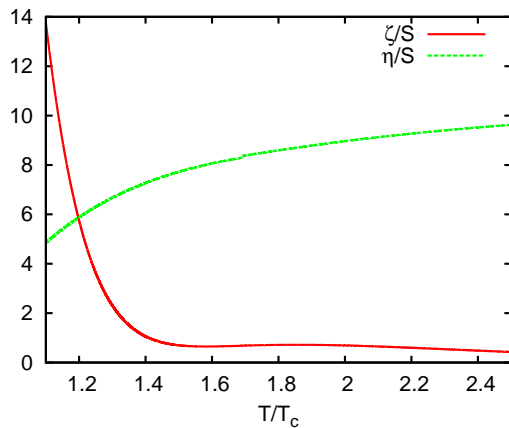


FIG. 6. (Color online) The temperature dependence of the quantities, η/S and ζ/S in the full QCD. Here $S = \frac{ST^3}{q}$, where S denotes the entropy density.

portional to the cross-sections. The bulk viscosity of full QCD is found to be equally significant as the shear viscosity while modeling the QGP. Indications are seen to a blow up in the bulk viscosity as we go closer to T_c .

The ratio, ζ/η which has been determined unambiguously in our approach, suggests that the QGP becomes almost conformal around $1.4T_c - 1.5T_c$. The ratio sharply decreases from $T = 1.1T_c - 1.4T_c$, and beyond that slowly approaches to zero. Therefore, in this regime we can ignore the effects of ζ while studying the hydrodynamic evolution and properties of the QGP. We further found that η and ζ are of same order around $T = 1.2T_c$. For temperatures lower than that ζ is dominant and for higher temperature η is dominant. Importantly, both η

and ζ came out to be highly sensitive to the presence of interactions. This can be visualized from the modulation of η , as compared to its ideal counter part, and large and rising value of ζ for the temperatures that are closer to T_c (due to large interaction measure there).

The investigations on the other contribution to the shear and bulk viscosities (collisional *etc.*), and their interplay with the corresponding anomalous transport coefficients will be a matter of future investigations. It will be interesting to include the effects of non-vanishing baryon density to the transport coefficients of the QGP. Moreover, one could include the anomalous transport coefficients in the Boltzmann-transport theory approach and study the impact on the response functions and quarkonia physics along the lines of [69, 70], as well as dilepton production at the RHIC and the LHC. These ideas will be studied in the near future.

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